Archimedes Principle

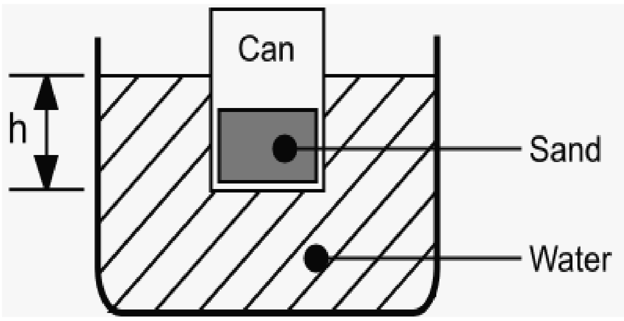
Objectives-

1- To measure gauge pressure.
2- To verify Archimedes' Principle.
3- To use this principle to
- determine the density of an unknown liquid
- determine the density of an irregular solid
- Specific gravity of various objects and fluids .

Theory:-

Part I. The Pressure–Depth Relation:

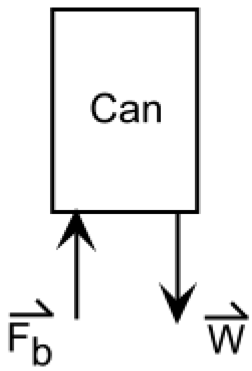
A body, which is less dense than water, placed on a water surface will sink into the liquid until the body experiences a buoyant force, *B* that equals its weight, *W*. This means that when the body floats, its weight and the buoyant force are the same in magnitude but opposite in direction (sound familiar?). You will use a cylinder (an aluminum can) so that the buoyant force due to the fluid acts only on the bottom of the cylinder if the can floats vertically.



Once you know the force which acts on the bottom of the can and the area of the bottom you can find the **pressure** on the bottom of the can. This is a *gauge* pressure because it assumes that the downward force is due only to the weight of the can and that the atmosphere makes no contribution. From the definition of pressure, we have:

$$\text{Pr}essure = \frac{Force}{Area} = \frac{F}{A} = \frac{W}{A} = \frac{mg}{A} \quad \text{........(1)}$$

$$\left\{ units: \frac{N}{m^2} = pascal \right\}$$



This gauge pressure is the pressure of the water on the cylinder bottom at that depth below the surface.

We can also use the pressure-depth relation to calculate the pressure some distance below the fluid surface:

$$p = p_0 + \rho_f gh$$

$$p - p_0 = \rho_f gh$$

$$\Delta p = \rho_f gh \quad \text{...............(2)}$$

Where ρ_f is the density of fluid and *h* is the depth in the liquid.

Part II. Archimedes Principle:

A) Verification of Archimedes Principle:

The buoyant force is described by Archimedes' principle as: *an object, when placed in a fluid, is buoyed up by a force equal to the weight of the fluid displaced by the object.* The principle applies to an object either entirely or partially submerged in the fluid. The magnitude of the buoyant force depends *only* on the weight of the displaced fluid, and not on the object's weight.

If a object is held at rest by a string (in air), its weight is given by

$$(W_b)_{air} = m_b g = \rho_b V_b g \qquad \text{……………………. (3)}$$

Where m_b, ρ_b, V_b are the mass, density and volume of the object, respectively.

If a solid is submerged in a fluid, it will be acted upon by three forces.

1. The weight of the body, W_b.
2. The buoyant force, B, on the body, which can be similarly expressed using Archimedes Principle:

$$B = W_f = m_f g = \rho_f V_f g, \text{…………… (4)}$$

where the subscript f refers to the fluid.

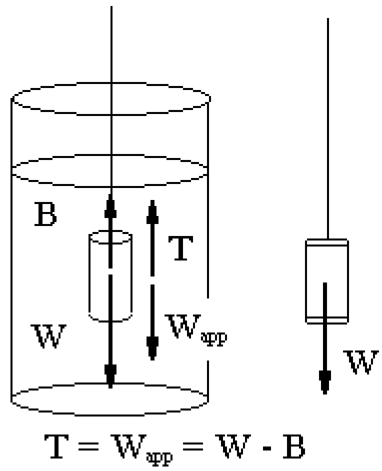
3. The tension in the string, T = apparent weight $W_{app.}$

Since the body is in equilibrium, $T(W_{app.}) + B - W_b = 0$.

Obviously, we can compute the buoyant force as

$$B = W_b - W_{app.} \text{ …………….. (5)}$$

i.e., from the difference of the actual weight of the body in air and the apparent weight of the body in the fluid.



B) Density of Unknown Liquid

Also from Archimedes principle, we can deduce the density of unknown liquid.

From equation (5)

$$W_{app.} = W_b - B$$

Where V_o is the volume of the submerged part of the object.

The volume of the submerged part of a cuboid oriented vertically is equal to its cross-sectional area A multiplied by the height h of the submerged part, so

$$W_{app.} = mg - (A\rho_f g)h \text{ ……………(6)}$$

This is a linear relationship between $W_{app.}$ and h, the slope of the plotted straight line will be $A\rho_f g$.

C) Specific gravity

Specific gravity (S.G.) of any substance is the ratio of the density of a body to the density of some standard substance. Within the limits of accuracy of this exercise, water at room temperature ρ_f may be chosen as the standard

$$S.G. = \frac{\rho_b}{\rho_f}$$

Since the volume of the water is necessarily equal to the volume of the body immersed (call them *V*), then

$$S.G = \frac{\rho_b}{\rho_f} = \frac{\rho_b V g}{\rho_f V g} = \frac{W_b}{B} = \frac{W_b}{W_b - W_{app}}$$

$$S.G = \frac{m_b}{m_b - m_{app.}} \text{............................}(7)$$

D) Density of irregular Solid

You can determine the density of an unknown solid from equation

$$\rho_b = \frac{m_b}{V_b} \text{.....................}(8)$$

It's easy to measure the mass of an object, but unless it has a regular shape it's not so easy to measure its volume. But Archimedes showed us how to measure volume by measuring weight.

This upward force is equal to the weight of the displaced fluid. But the volume of the fluid is equal to the volume of the object. From equation (4) and (8) the density of body given by

$$\rho_b = \frac{m_b}{V_f} = \frac{m_b \rho_f g}{B} = \frac{m_b \rho_f g}{W_b - W_{app}}$$

$$\rho_b = \frac{m_b \rho_f}{m_b - m_{app}} \text{.......................... }(9)$$

<u>Apparatus:</u>-

Balance, container of cork, sand, beaker, graduated cylinder, cuboids, sinker, unknown fluid, string, and Venire caliper.

<u>Procedure:</u>-

Part I. The Pressure–Depth Relation:

1. Measure the dimensions cork container base.
2. Load a cork container with sand so that it floats in the water and shake the sand about until the cork floats upright and level. Tilt the cork to allow any air trapped beneath it to escape.

3. Measure the depth below the water surface of the *bottom* of the cork, h.
4. Remove the cork from the water, dry it off, and measure its mass. Then calculate the gauge pressure at depth *h* from equation (1), and the equation (2)
5. Tabulate your measurement in table 1

Part II. Archimedes Principle:

A) Verification of Archimedes Principle:

1- .Use a vernier caliper (and/or) micrometer to measure the dimensions of the object (cuboids). So, calculate the volume of body V_b which is the same volume of fluid displaced V_f..

2- Mark off the cuboids every 1 cm vertically starting from the bottom.

3- Suspend the cuboids by string from the weigh-below hook without touching the empty beaker located on platform

4- Determine the mass of the cuboids using a balance (m_b), so (w_b).

5- Now pour fluid from another beaker slowly from the side. Fill the beaker to a level matching the first of your marks.

6- Record the new weight. Repeat for the next mark.

7- Fill the beaker of fluid so that the sample is completely submerged without touching the container and measure the m_{app}. and $W_{app.}$.

8- Tabulate your measurement in table 2.

9- Calculate buoyant force B_1 from equation (4), and another way B_2 such as equation (5) and compare between B1 and B_2.

B) Density of Unknown Liquid

1. Using the measurement of table 2.
2. Plot the graph of appearance weight $W_{app.}$ as vertical axis against h as horizontal axis, then determine the ρ_f of the fluid by equation (6).

C) Specific gravity

1. Using the measurement of table 2.
2. Substitute in equation(7).

D) Density of irregular Solid (Density of Rock)

1- Suspend the irregular body (**Rock**) in balance by string and measure the(m_b).

2- But the known fluid density (Water is a convenient liquid to use because its density equals 998 kg/m^3) in the graduated cylinder and read the volume(V_f)

3- Immersed object in the graduated cylinder. Notice fluid with rise record the volume (V_{f+b}) and mass m_{app}.

4- Calculate the volume of object (V_b) from $V_b = V_{f+b} - V_b$. used equation (8) to calculate the $(\rho_b)_1$

5- Calculate density of body $(\rho_b)_2$ by another method, such as equation (9).

6- Record the data in table 3 and compare between $(\rho_b)_1$ and $(\rho_b)_2$.

Measurements and result:-

Part I. The Pressure–Depth Relation

Table 1

Floating sample	Base dimensions L*W	Base area A (m^2)	Mass m (Kg)	Weight W (N)	$P_1 = \frac{W}{A}$ (Pa)	$P_2 = \rho g h$ (Pa)	%Eror $= \frac{p_2 - p1}{p_1}$ *100
1							
2							

Part II. Archimedes Principle:

Table 2

m_b=……………………Kg.　　　　W_b= m_b* g =……………………N

Base area of cuboids (A) =……………*……………=……..……………m^2

Theoretical density of Water ρ_f= 998 Kg/m^3

Height (h) (m)	**$m_{app.}$ (Kg)**	**Wapp (N)**
Total submerge(h_t)		

A) Verification of Archimedes Principle:

V_b=V_f=A*h_t=…………………………………………m^3

$B_1 = \rho_f V_f g =$ ……………………………………..N

$B_2 = W_b - W_{app.} =$ …………………………………..N

B) Density of Unknown Liquid

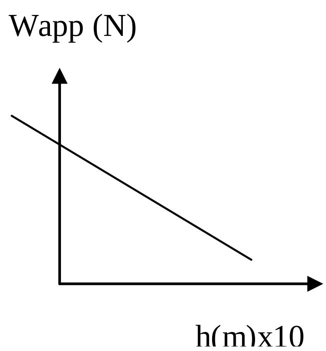
$$W_{app.} = mg - (A\rho_f g)h$$

From graph

$W_{app.}$ V.I.=………………………………..N

Slope=………………………………N/m

$\rho = \dfrac{slope \times g}{A} =$ ……………………..Kg/m^3



C) Specific gravity

From table 2

$S.G = \dfrac{m_b}{m_b - m_{app.}}$ =……………………….=……………………..

D) Density of irregular Solid (Density of Rock)

Table 3

$\rho_f = \rho_w$ = 998 Kg/m3

m_{app} (Kg)	V_f (m^3)	V_{f+b} (m^3)	V_b (m^3)	$(\rho_b)1 = \frac{m_b}{V_b}$ (Kg/m^3)	$(\rho_b)2 = \frac{m_b \rho_f}{m_b - m_{app}}$ (Kg/m^3)